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Scheduling over scenarios

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Apart from some exceptions, modeling uncertainty makes problems harder. Scenarios are basic ingredients of optimization problems under uncertainty. Usually, they are specified only implicitly, e.g. as ranges over parameter values leading to robust optimization. Or with a distribution function over the scenarios, leading to stochastic optimization problems. Dealing with scenarios becomes more and more important. A classical model in algorithm design here is the stochastic linear optimization problem. A popular solution method is the Sample Average Algorithm, which samples scenarios and optimizes over them as an approximation. In modeling the value of data is more and more recognized, which predicts scenarios that occur in problems more or less frequently. Anyway, if in these applications scenarios are specified completely and individually, then the input size increases significantly, and therefore the computational complexity of the problems may not increase, like is the case with stochastic linear optimization.

In this lecture we explore how introduction of fully specified scenarios may alter the complexity of problems. We do so at the hand of studying, most prominently, basic scheduling problems. In this problem we are given a set J of jobs, each with its processing time, a set of parallel identical machines and a set of k scenarios, where each scenario is specified as a subset of jobs in J that must be executed if that scenario occurs. The goal is to find an assignment of jobs to the machines that is the same for all scenarios, i.e., if a job does not occur in a scenario it is simply skipped. We consider the classical problems of minimizing the makespan and minimizing the sum of the jobs' completion times. For each we consider the objectives of minimizing the maximum over all scenarios (robust version) and minimizing the average of makespans of all scenarios (stochastic version).

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We show that the presence of scenarios may increase the complexity of the problems significantly. As a dramatic example, I mention the ordinary, single scenario, version of the makespan problem with all processing times equal to 1, which is a trivial problem. With scenarios the robust version of the problem becomes inapproximable in polynomial time within ratio 2 unless $P = NP$. This is even more surprising once one realizes that putting al jobs on one machine already yields a 2-approximation.

Similar results hold for the sum of completion times problem. As for the makespan problem, the leap in complexity requires that the number of scenarios is part of the input. However, I will give an example of a scheduling problem minimizing total completion time that is in P in its ordinary version and becomes NP-hard already for 3 scenarios. These results mostly show the mysterious role that scenarios play in the complexity of combinatorial optimization problems.

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